Pre-Activity  How are translations used in a marching band show?

Read the introduction to Lesson 9-2 at the top of page 470 in your textbook.

How do band directors get the marching band to maintain the shape of the figure they originally formed?

Reading the Lesson

1. Underline the correct word or phrase to form a true statement.
   a. All reflections and translations are (opposites/isometries/equivalent).
   b. The preimage and image of a figure under a reflection in a line have (the same orientation/opposite orientations).
   c. The preimage and image of a figure under a translation have (the same orientation/opposite orientations).
   d. The result of successive reflections over two parallel lines is a (reflection/rotation/translation).
   e. Collinearity (is/is not) preserved by translations.
   f. The translation \( (x, y) \rightarrow (x + a, y + b) \) shifts every point \( a \) units (horizontally/vertically) and \( y \) units (horizontally/vertically).

2. Find the image of each preimage under the indicated translation.
   a. \((x, y); 5 \text{ units right and 3 units up}\)
   b. \((x, y); 2 \text{ units left and 4 units down}\)
   c. \((x, y); 1 \text{ unit left and 6 units up}\)
   d. \((x, y); 7 \text{ units right}\)
   e. \((4, -3); 3 \text{ units up}\)
   f. \((-5, 6); 3 \text{ units right and 2 units down}\)
   g. \((-7, 5); 7 \text{ units right and 5 units down}\)
   h. \((-9, -2); 12 \text{ units right and 6 units down}\)

3. \(\triangle RST\) has vertices \(R(-3, 3), S(0, -2), \text{ and } T(2, 1)\). Graph \(\triangle RST\) and its image \(\triangle R'S'T'\) under the translation \( (x, y) \rightarrow (x + 3, y - 2) \). List the coordinates of the vertices of the image.

Helping You Remember

4. A good way to remember a new mathematical term is to relate it to an everyday meaning of the same word. How is the meaning of translation in geometry related to the idea of translation from one language to another?
You can use algebraic descriptions of reflections to show that the composite of two reflections with respect to parallel lines is a translation (that is, a slide).

1. Suppose \( a \) and \( b \) are two different real numbers. Let \( S \) and \( T \) be the following reflections:

\[
S: (x, y) \rightarrow (-x + 2a, y)
\]

\[
T: (x, y) \rightarrow (-x + 2b, y)
\]

\( S \) is reflection with respect to the line with equation \( x = a \), and \( T \) is reflection with respect to the line with equation \( x = b \).

a. Find an algebraic description (similar to those above for \( S \) and \( T \)) to describe the composite transformation “\( S \) followed by \( T \).”

b. Find an algebraic description for the composite transformation “\( T \) followed by \( S \).”

2. Think about the results you obtained in Exercise 1. What do they tell you about how the distance between two parallel lines is related to the distance between a preimage and image point for a composite of reflections with respect to these lines?

3. Illustrate your answers to Exercises 1 and 2 with sketches. Use a separate sheet if necessary.